

Properties of Logic and Boolean Algebra

1 Basic Logical Equivalencies

1. Absorption Law: $p \vee (p \wedge q) \equiv p$ and $p \wedge (p \vee q) \equiv p$
2. Associative Law: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ and $(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Commutative Law: $p \wedge q \equiv q \wedge p$ and $p \vee q \equiv q \vee p$
4. Contradiction: $c \equiv \neg t$
5. DeMorgan's Law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$ and $\neg(p \vee q) \equiv \neg p \wedge \neg q$
6. Distributive Law: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ and $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
7. Double Negative: $\neg(\neg p) \equiv p$
8. Idempotent Law: $p \wedge p \equiv p$ and $p \vee p \equiv p$
9. Identity Law: $p \wedge t \equiv p$ and $p \vee c \equiv p$
10. Material Equivalence: $p \iff q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$ and $p \iff q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
11. Material Implication: $p \Rightarrow q \equiv \neg p \vee q$
12. Negation Law: $p \wedge \neg p \equiv c$ and $p \vee \neg p \equiv t$
13. Tautology: $t \equiv \neg c$
14. Universal Bounds: $p \vee t \equiv p$ and $p \wedge c \equiv p$

2 Valid Arguments

RULE	PREMISES	CONCLUSION	PREMISES	CONCLUSION
Absorption	$p \Rightarrow q$	$\therefore p \Rightarrow p \wedge q$		
Conjunction/Adjunction	p, q	$\therefore p \wedge q$		
Constructive Dilemma	$(p \Rightarrow q) \wedge (r \Rightarrow s), p \wedge r$	$\therefore q \wedge s$		
Contradiction	$\neg p \Rightarrow c$	$\therefore p$		
Elimination/Modus Tollendo Ponens	$p \vee q, \neg p$	$\therefore q$	$p \vee q, \neg q$	$\therefore p$
Generalization/Addition	p	$\therefore p \vee q$	q	$\therefore p \vee q$
Modus Ponens	$p \Rightarrow q, p$	$\therefore q$		
Modus Tollens	$p \Rightarrow q, \neg q$	$\therefore \neg p$		
Specialization/Simplification	$p \wedge q$	$\therefore p$	$p \wedge q$	$\therefore q$
Transitivity/Hypothetical Syllogism	$p \Rightarrow q, q \Rightarrow r$	$\therefore p \Rightarrow r$		