Deriving Logical Conclusions

July 14, 2009

Using Valid Arguments, derive the conclusion from the given premises.

Premises: $p \vee q$, $q \Rightarrow r$, $p \wedge s \Rightarrow t$, $\neg r$, $\neg q \Rightarrow u \wedge s$.

Conclusion: $\therefore t$

Let's start with the premises $\neg r$ and $q \Rightarrow r$. By Modus Tollens, we have the conclusion $\therefore \neg q$. Using the original premise $\neg q \Rightarrow u \wedge s$ and our new premise, $\neg q$, we have the conclusion $\therefore u \wedge s$ by Modus Ponens. Using our new premise, $u \wedge s$, we have the conclusion $\therefore s$ by Specialization. Using the premises $p \vee q$ and $\neg q$, we have the conclusion $\therefore p$ by Elimination. Using our new premises, p and s, we have the conclusion $\therefore p \wedge s$ by Conjunction. Finally, using our new premise, $p \wedge s$, and the original premise, $p \wedge s \Rightarrow t$, we have derived the conclusion $\therefore t$ by Modus Ponens.

Premises: $(\neg s \wedge \neg u) \Rightarrow v$, $q \vee \neg t$, $p \Rightarrow q$, $(\neg p \wedge \neg q \wedge \neg t) \Rightarrow \neg s$, $\neg q$, $\neg u$.

Conclusion: $\therefore v$

Let's start with the premises $\neg q$ and $p \Rightarrow q$. By Modus Tollens, we have the conclusion $\therefore \neg p$. Using the premises $q \vee \neg t$ and $\neg q$, we have the conclusion $\therefore \neg t$ by Elimination. Using the new premises $\neg t$ and $\neg p$, as well as the original premise, $\neg q$, we have the conclusion $\therefore (\neg p \wedge \neg q \wedge \neg t)$ by Conjunction. Using the new premise $(\neg p \wedge \neg q \wedge \neg t)$, and our original premise $(\neg p \wedge \neg q \wedge \neg t) \Rightarrow \neg s$, we have the conclusion $\therefore \neg s$ by Modus Ponens. Using our new premise, $\neg s$, and our original premise $\neg u$, we have the conclusion $\therefore \neg s \wedge \neg u$ by Conjunction. Finally, using our new premise, $\neg s \wedge \neg u$, and the original premise, $(\neg s \wedge \neg u) \Rightarrow v$, we have derived the conclusion $\therefore v$ by Modus Ponens.

Premises: $(\neg p \vee q) \Rightarrow r$, $s \vee \neg q$, $\neg t$, $p \Rightarrow t$, $(\neg p \wedge r) \Rightarrow \neg s$.

Conclusion: $\therefore \neg q$

Let's start with the premises $p \Rightarrow t$ and $\neg t$. By Modus Tollens, we have the conclusion $\therefore \neg p$. Using the new premise, $\neg p$, we have the conclusion $\therefore (\neg p \vee q)$ by Generalization. Using the original premise $(\neg p \vee q) \Rightarrow r$ and the new premise $(\neg p \vee q)$, we have the conclusion $\therefore r$ by Modus Ponens. Using the new premises $\neg p$ and r, we have the conclusion $\therefore (\neg p \wedge r)$ by Conjunction. Using the new premise $(\neg p \wedge r)$, and our original premise $(\neg p \wedge r) \Rightarrow \neg s$, we have the conclusion $\therefore \neg s$ by Modus Ponens. Using our new premise, $\neg s$ and our original premise $s \vee \neg q$, we have derived the conclusion $\therefore \neg q$ by Elimination.

Premises: $\neg p \Rightarrow r \wedge \neg s$, $t \Rightarrow s$, $u \Rightarrow \neg p$, $\neg w$, $u \vee w$.

Conclusion: $\therefore \neg t$

Let's start with the premises $u \vee w$ and $\neg w$. By Elimination, we have the conclusion $\therefore u$. Using the new premise, u, and the original premise $u \Rightarrow \neg p$, we have the conclusion $\therefore \neg p$ by Generalization. Using the new premise, $\neg p$, and the original premise, $\neg p \Rightarrow r \wedge \neg s$, we have the conclusion $\therefore r \wedge \neg s$ by Modus Ponens. Using the premise $r \wedge \neg s$, we have the conclusion $\therefore \neg s$ by Specialization. Using the new premise $\neg s$, and our original premise $t \Rightarrow s$, we we have derived the conclusion $\therefore \neg t$ by Elimination.