Proving Logical Equivalencies

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Prove that $\neg(\neg p \vee (\neg p \wedge \neg q)) \equiv p$.

By the DeMorgan's Law, we have that $\neg(\neg p \vee (\neg p \wedge \neg q)) \equiv \neg(\neg p \vee \neg(p \vee q)) \equiv \neg(\neg p) \wedge \neg(\neg(p \vee q))$. By Double Negation, we obtain $p \wedge (p \vee q)$. By the Absorption Law, we obtain p as desired.

Prove that $(p \wedge q) \wedge \neg(\neg p \vee q) \equiv p$.

By DeMorgan's Law, we have that $(p \wedge q) \wedge \neg(\neg p \vee q) \equiv (p \wedge q) \wedge (\neg(\neg p) \wedge \neg q)$. By Double Negation, we obtain $(p \wedge q) \wedge (p \wedge \neg q) \equiv p \wedge q \wedge p \wedge \neg q$. By the Commutative Law, we obtain $p \wedge p \wedge q \wedge \neg q \equiv (p \wedge p) \wedge (q \wedge \neg q)$. By the Idempotent Law, we obtain $p \wedge (q \wedge \neg q)$. By the Negation Law, we obtain $p \wedge c$. By the Universal Bounds Law, we obtain p as desired.

Prove that $(p \wedge \neg q) \wedge \neg(q \wedge \neg r) \equiv p \wedge \neg q$.

By DeMorgan's Law, we have that $(p \wedge \neg q) \wedge \neg(q \wedge \neg r) \equiv (p \wedge \neg q) \wedge (\neg q \vee \neg(\neg r))$. By Double Negation, we obtain $(p \wedge \neg q) \wedge (\neg q \vee r)$. By Distributive Law, we obtain $((p \wedge \neg q) \wedge \neg q) \vee ((p \wedge \neg q) \wedge r)$. By the Associative Law, we obtain $(p \wedge (\neg q \wedge \neg q)) \vee ((p \wedge \neg q) \wedge r)$. By the Idempotent Law, we obtain $(p \wedge \neg q) \vee ((p \wedge \neg q) \wedge r)$. By the Absorption Law, we obtain $p \wedge \neg q$ as desired.

Prove that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$.

By DeMorgan's Law, we have that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv \neg((\neg p \wedge q) \vee \neg(p \vee q)) \vee (p \wedge q) \equiv (\neg(\neg p \wedge q) \vee \neg(\neg(p \vee q))) \vee (p \wedge q) \equiv (\neg(\neg p) \vee \neg q) \vee \neg(\neg(p \vee q))) \vee (p \wedge q)$. By Double Negation, we obtain $((p \vee \neg q) \vee (p \vee q)) \vee (p \wedge q) \equiv (p \vee \neg q \vee p \vee q) \vee (p \wedge q)$. By Commutative Law, we obtain $(p \vee p \vee \neg q \vee q) \vee (p \wedge q) \equiv ((p \vee p) \vee (\neg q \vee q)) \vee (p \wedge q)$. By the Idempotent and Negation Laws, we obtain $(p \vee t) \vee (p \wedge q)$. By the Universal Bounds Law, we obtain $p \vee (p \wedge q)$. By the Absorption Law, we obtain p as desired.

Prove that $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q)) \equiv \neg p$.

By DeMorgan's Law, we have that $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q)) \equiv \neg(p \vee \neg q) \vee \neg(p \vee q) \equiv \neg((p \vee \neg q) \wedge (p \vee q))$. By Distributive Law, we obtain $\neg(((p \vee \neg q) \wedge p) \vee ((p \vee \neg q) \wedge q)) \equiv \neg(((p \wedge p) \vee (p \wedge \neg q)) \vee ((p \wedge q) \vee (q \wedge \neg q)))$. By Idempotent Law, we obtain $\neg((p \vee (p \wedge \neg q)) \vee ((p \wedge q) \vee (q \wedge \neg q)))$. By Universal Bounds Law, we obtain $\neg(p \vee (p \wedge \neg q)) \vee ((p \wedge q) \vee c))$. By Absorption Law, we obtain $\neg(p \vee ((p \wedge q) \vee c))$. By Identity Law, we obtain $\neg(p \vee (p \wedge q))$. By Absorption Law, we obtain $\neg p$ as desired.