Proving Set Identities

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Let X be the universe. Prove that $A \cup (B \setminus A) = A \cup B$.

By the Set Difference Law, we have that $A \cup (B \setminus A) = A \cup (B \cap A^{\complement})$. By the Commutative Law, we obtain $A \cup (A^{\complement} \cap B)$. By the Distributive Law, we obtain $(A \cup A^{\complement}) \cap (A \cup B)$. By the Complement Law, we obtain $X \cap (A \cup B)$. By the Identity Law, we obtain $A \cup B$ as desired.

Let X be the universe. Prove that $(A \setminus B) \setminus C = A \setminus (B \cup C)$.

By the Set Difference Law, we have that $(A \setminus B) \setminus C = (A \cap B^{\complement}) \setminus C = (A \cap B^{\complement}) \cap C^{\complement}$. By the Associative Law, we obtain $A \cap (B^{\complement} \cap C^{\complement})$. By DeMorgan's Law, we obtain $A \cap (B \cup C)^{\complement}$. By the Set Difference Law, we obtain $A \setminus (B \cup C)$.

Let X be the universe. Prove that $(B^{\complement} \cup (B^{\complement} \setminus A))^{\complement} = B$.

By the Set Difference Law, we have that $(B^{\complement} \cup (B^{\complement} \setminus A))^{\complement} = (B^{\complement} \cup (B^{\complement} \cap A^{\complement}))^{\complement}$. By the DeMorgan's Law, we obtain $(B^{\complement} \cup (B \cup A)^{\complement})^{\complement} = B \cap (B \cup A)$. By Absorption Law, we obtain B as desired.

Let X be the universe. Prove that $A \setminus (A \setminus B) = A \cap B$.

By the Set Difference Law, we have that $A \setminus (A \setminus B) = A \setminus (A \cap B^{\complement}) = A \cap A \cap B^{\complement^{\complement}}$. By DeMorgan's Law, we obtain $A \cap (A^{\complement} \cup B)$. By the Distributive Law, we obtain $(A \cap A^{\complement}) \cup (A \cap B)$. By the Complement Law, we obtain $\emptyset \cup (A \cap B)$. By the Identity Law, we obtain $A \cap B$ as desired.

Let X be the universe. Prove that $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$.

By the Set Difference Law, we have that $(A \setminus B) \cup (B \setminus A) = (A \cap B^{\complement}) \cup (B \cap A^{\complement})$. By Distributive Law, we obtain $((A \cap B^{\complement}) \cup B) \cap ((A \cap B^{\complement}) \cup A^{\complement}) = ((A \cup B) \cap (B \cup B^{\complement})) \cap ((A \cup A^{\complement}) \cap (B^{\complement} \cup A^{\complement}))$. By the Complement Law, we obtain $((A \cup B) \cap X) \cap (X \cap (B^{\complement} \cup A^{\complement}))$. By the Commutative Law and DeMorgan's Law, we obtain $((A \cup B) \cap X) \cap (X \cap A \cap B^{\complement})$. By the Identity Law, we obtain $(A \cup B) \cap (A \cap B^{\complement})$. By Set Difference Law, we obtain $(A \cup B) \setminus (A \cap B)$ as desired.