Properties of Sets

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1 Set Equivalencies

Let X be the universe and let A and B be two arbitrary, nonempty sets such that $A \subseteq X$ and $B \subseteq X$. Thus:

1. Absorption Law: $A \cup (A \cap B) = A$ and $A \cap (A \cup B) = A$
2. Associative Law: $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$
3. Commutative Law: $A \cup B = B \cup A$ and $A \cap B = B \cap A$
4. Complement Law: $A \cup A^{\complement} = X$ and $A \cap A^{\complement} = \emptyset$
5. DeMorgan's Law: $(A \cap B)^{\complement} = A^{\complement} \cup B^{\complement}$ and $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$
6. Distributive Law: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
7. Double Complement Law: $(A^{\complement})^{\complement} = A$
8. Empty Set: $\emptyset = X^{\complement}$
9. Idempotent Law: $A \cup A = A$ and $A \cap A = A$
10. Identity Law: $A \cup \emptyset = A$ and $A \cap X = A$
11. Set Difference Law: $A \setminus B = A \cap B^{\complement}$
12. Universal Bounds Law: $A \cup X = X$ and $A \cap \emptyset = \emptyset$
13. Universal Set: $X = \emptyset^{\complement}$

2 Properties of Cartesian Products of Sets

1. Distributive Law 1: $A \times (B \cap C) = (A \times B) \cap (A \times C)$ and $(B \cap C) \times A = (B \times A) \cap (C \times A)$
2. Distributive Law 2: $A \times (B \cup C) = (A \times B) \cup (A \times C)$ and $(B \cup C) \times A = (B \times A) \cup (C \times A)$
3. $A \times \emptyset = \emptyset$ and $\emptyset \times A = \emptyset$
4. $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

3 Basic Subset Relations

1. Inclusion of Intersection: $A \cap B \subseteq A$ and $A \cap B \subseteq B$
2. Inclusion of Union: $A \subseteq A \cup B$ and $B \subseteq A \cup B$
3. Transitive Property: If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

4 Procedural Version of Set Definition

1. $x \in A \cup B \iff x \in A$ or $x \in B$
2. $x \in A \cap B \iff x \in A$ and $x \in B$
3. $x \in A \setminus B \iff x \in A$ but $x \notin B$
4. $x \in A^{\complement} \iff x \notin A$