Closed Forms of Summations

(all of these can be proven by Mathematical Induction)

1. $\sum_{i=0}^{n} a^i = \frac{a^{n+1}-1}{a-1}$
2. $\sum_{i=0}^{n} i(i!) = (n+1)! - 1$
3. $\sum_{i=0}^{n} \frac{i}{(i+1)!} = \frac{(n+1)!-1}{(n+1)!}$
4. $\sum_{i=0}^{n} ai = \frac{a[n(n+1)]}{2}$
5. $\sum_{i=0}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$
6. $\sum_{i=0}^{n} i(i+1)(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
7. $\sum_{i=0}^{n} (2i+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$
8. $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
9. $\sum_{i=0}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$
10. $\sum_{i=0}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
11. $\sum_{i=0}^{n} \frac{1}{2^i} = \frac{2^{n+1}-1}{2^n}$
12. $\sum_{i=0}^{n} \frac{i}{2^i} = 2 - \frac{n+2}{2^n}$
13. $\sum_{i=1}^{n} (i+1)2^n = n2^{n+1}$
14. $\sum_{i=1}^{n} (2i-1)^3 = n^2(2n^2-1)$
15. $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$
16. $\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$
17. $\sum_{i=1}^{n} (2i-1) = n^2$
18. $\sum_{i=1}^{n} \frac{1}{i(i+1)(i+2)} = \frac{n^2+3n}{4(n+1)(n+2)}$
19. $\sum_{i=2}^{n} \frac{1}{(i-1)(i+1)} = \frac{3n^2-n-2}{4n(n+1)}$